Errata for

Dodge, Franklin T., ***THE NEW "DYNAMIC BEHAVIOR OF LIQUIDS IN MOVING CONTAINERS",*** Southwest Research Institute, San Antonio, Texas, 2000

References:

Abramson, H. N., "Dynamic Behavior of Liquids in Moving Containers," NASA SP-106, 1966.

Bauer, H., "Fluid Oscillations in the Containers of a Space Vehicle and Their Influence Upon Stability," NASA TR R-187, 1964.

<table>
<tr><td>1</td><td>Equation 1.8 is shown as
$$n \cdot \nabla\Phi = iX_o\Omega e^{i\Omega t}$$
To be consistent with the text in the previous paragraph, Equation 1.9 should read as
$$n \cdot \nabla\Phi = X_o\Omega e^{i\Omega t}$$</td></tr>
<tr><td>2</td><td>Equation 1.25 is shown as
$$x = \sum_{n=1}^{\infty}\left(\frac{2a^2}{\lambda_n^2}\right)(-1)^{n-1}\sin\left(\lambda_n x/a\right)$$
The first term after the summation symbol is incorrect. Equation 1.25 should read as
$$x = \sum_{n=1}^{\infty}\left(\frac{4a}{\lambda_n^2}\right)(-1)^{n-1}\sin\left(\lambda_n \frac{x}{a}\right)$$</td></tr>
<tr><td>3</td><td>Equation 1.26, the coefficient A should be A_o as defined above as $A_o = X_o\Omega$ in Eq. 1.24</td></tr>
<tr><td>4</td><td>Section 1.4 is intended to show the eigenvalues and only the <u>sloshing</u> part of the overall eigensolutions to the fully cylindrical tank as a special case of the general ring-sector velocity potential equation. Equation 1.40 is shown as
$$\Phi_{m,n}(r,z) = J_1\left(\frac{\lambda_{mn} r}{a}\right)\cos(m\theta)\frac{\cosh[\lambda_{mn}(z/a + h/2a)]}{\cosh[\lambda_n h/a]}$$
This equation is incorrect. To be consistent with SP-106 and Bauer. Equation 1.40 should read as
$$\Phi_n(r,\theta,z,\Omega) = \frac{2J_1\left(\frac{\lambda_n r}{a}\right)\cos(\theta)\cosh\left[\lambda_n\left(\frac{z}{a}+\frac{h}{a}\right)\right]\left(\frac{\Omega}{\omega_n}\right)^2}{(\lambda_n^2 - 1)J_1(\lambda_n)\cosh\left[\frac{\lambda_n h}{a}\right]\left(1-\left(\frac{\Omega}{\omega_n}\right)^2\right)}$$
For a non-sectored circular tank, the mode m=2 of the general solution (see Eq. 1.41) is the only circumferential mode of the general solution that leads to non-zero forces and moments on the tank wall. So, only the radial 'n' modes are represented in Equation 1.40. The eigenvalues for these modes are the solutions of $J'_1(\lambda_n) = 0$.
The corresponding eigenfrequencies are
$$\lambda_n^2 = \frac{g}{a}\tanh\left[\lambda_n \frac{h}{a}\right]$$</td></tr>
</table>

5	In Section 1.4 “Antisymmetric modes”, the references to m=1 are inconsistent with the nomenclature used in SP-106 and Bauer. These should be changed to m=2 and m>2 as appropriate. C
6	Section 1.5 is intended to show the eigenvalues and only the <u>sloshing</u> part of the overall eigensolutions to the general ring-sector velocity potential equation. Equation 1.41a is shown as $$\Phi_{m,n}(r,\phi,z) = \cos\left(\frac{m\theta}{2\alpha}\right)\frac{\cosh[\lambda_{mn}(z/a+h/2a)]}{\cosh[\lambda_{mn}\,h/a]}\chi(\lambda_{mn}\,r/a)$$ This equation is incorrect. To be consistent with SP-106 and Bauer. Equation 1.41a should read as $$\Phi_{m,n}(r,\theta,z,\Omega) = \cos\left(\frac{m\theta}{2\alpha}\right)\frac{a_m b_{mn}\cosh\left[\lambda_{mn}\left(\frac{z}{a}+\frac{h}{a}\right)\right]\left(\frac{\Omega}{\omega_{mn}}\right)^2}{\cosh\left[\frac{\lambda_{mn}h}{a}\right]\left(1-\left(\frac{\Omega}{\omega_{mn}}\right)^2\right)}\chi\left(\frac{\lambda_{mn}r}{a}\right)$$ The coefficients a_m and b_{mn} are intricate algebraic expressions and are not shown here for the sake of brevity. The reader is referred to Bauer and SP-106 for their complete definitions. Equation 1.41b is shown as $$\chi(\lambda_{mn}\,r/a) = J_{m/2\alpha}(\lambda_{mn}\,r/a)Y'_{m/2\alpha}(\lambda_{mn}) - J'_{m/2\alpha}(\lambda_{mn})Y_{m/2\alpha}(\lambda_{mn}\,h/a)$$ This equation is incorrect. The correct definition for the χ function, Equation 1.41b is $$\chi\left(\frac{\lambda_{mn}r}{a}\right) = J_{\frac{m}{2\alpha}}\left(\frac{\lambda_{mn}r}{a}\right)Y'_{\frac{m}{2\alpha}}(\lambda_{mn}) - J'_{\frac{m}{2\alpha}}(\lambda_{mn})Y_{\frac{m}{2\alpha}}\left(\frac{\lambda_{mn}r}{a}\right)$$ The existing Equations 1.41c and 1.41d are consistent with the correct versions of Equations 1.41a and 1.41b.
7	Equation 1.41d is shown as $$\omega_{mn}^2 = \frac{g\lambda_{mn}}{a}\tanh\left(\frac{\lambda_{mn}r}{a}\right)$$ Equation 1.41d should read as $$\omega_{mn}^2 = \frac{g\lambda_{mn}}{a}\tanh\left(\frac{\lambda_{mn}h}{a}\right)$$
8	The paragraph immediately after Equation 1.41d reads as “For a quarter sector tank (α=0.5, b=0). …” This sentence should read as “For a quarter sector tank (α=0.25, b=0). …”

9	Equation 2.7a is shown as $$\gamma = 0.83\sqrt{Re_1}\left[\tanh(1.84h/R)\left(1+2\frac{1-h/R}{\cosh(3.68h/R)}\right)\right]$$ Equation 2.7a should read as $$\gamma = 0.83\sqrt{Re_1}\left(\tanh(1.84h/R)\right)^{-1/4}\left(1+2\frac{1-h/R}{\sinh(3.68h/R)}\right)$$
10	Equation 2.12 is shown as $$\gamma = \frac{(4/3\pi)C_D C_\theta A_b f_d^3 \delta}{2(m_s/\rho)\Gamma^2}$$ The fraction in the numerator should be clarified as $$\gamma = \frac{(4/(3\pi))C_D C_\theta A_b f_d^3 \delta}{2(m_s/\rho)\Gamma^2}$$
11	Equation 2.14 is shown as $$\gamma = \frac{15(4/3\pi)^2 C_D C_\theta A_b f_d^{2.5}\sqrt{\delta w}}{2\sqrt{\pi}(m_s/\rho)\Gamma^2}$$ The fraction in the numerator should be clarified as $$\gamma = \frac{15(4/(3\pi))^2 C_D C_\theta A_b f_d^{2.5}\sqrt{\delta w}}{2\sqrt{\pi}(m_s/\rho)\Gamma^2}$$
12	In Eq. 3.4a, the first $\ddot{x}$ should be capitalized as $\ddot{X}$
13	Equation 3.9 is shown as $$\frac{M_{amp}}{i\Omega^2} = -\alpha_0\left[I_0 + m_0H_0^2 + \sum m_nH_n^2 + m_{liq}\sum\frac{m_n}{m_{liq}}\left(\frac{H_n^2\Omega^2 + 2H_ng + gh^2/\omega_n^2}{\omega_n^2-\Omega^2}\right)\right] - m_0X_0\sum\frac{m_n}{m_{liq}}\left(\frac{H_n\Omega^2+g}{\omega_n^2-\Omega^2}\right)$$ The numerator in the fourth term should be changed so that Equation 3.9 reads $$\frac{M_{amp}}{i\Omega^2} = -\alpha_0\left[I_0 + m_0H_0^2 + \sum m_nH_n^2 + m_{liq}\sum\frac{m_n}{m_{liq}}\left(\frac{H_n^2\Omega^2 + 2H_ng + g^2/\Omega^2}{\omega_n^2-\Omega^2}\right)\right] - m_0X_0\sum\frac{m_n}{m_{liq}}\left(\frac{H_n\Omega^2+g}{\omega_n^2-\Omega^2}\right)$$
14	The label of the ordinate of the graph in Figure 3.3 is shown as $$I_o + m_oH_o^2 + \Sigma m_n(H_n - L_n)^2/\left[m_{liq}\left(\frac{3}{16}d^2 + \frac{1}{12}h^2\right)\right]$$ All terms to the left of the division symbol are in the numerator and the 3/16 in the denominator is incorrect. The correct expression is as follows: $$[I_o + m_oH_o^2 + \Sigma m_n(H_n - L_n)^2]/\left[m_{liq}\left(\frac{1}{16}d^2 + \frac{1}{12}h^2\right)\right]$$

15	Equation A.12 is shown as $$b_n = \frac{\pi}{V\gamma_n}\int r^2\Phi^{\langle n\rangle}dr = \frac{\pi}{V\gamma_n}\sum_{j=1}^{10} C_k^{\langle n\rangle}R^2\phi_k dR = \frac{\pi a^3}{V\gamma_n}\sum_{k=1}^{10} C_k^{\langle n\rangle}B_{1k}$$ It should read $$b_n = \frac{\pi}{V\gamma_n}\int r^2\Phi^{\langle n\rangle}dr = \frac{\pi}{V\gamma_n}\sum_{j=1}^{10} C_k^{\langle n\rangle}\int r^2\phi_k dr = \frac{\pi a^3}{V\gamma_n}\sum_{k=1}^{10} C_k^{\langle n\rangle}B_{1k}$$
16	Equation A.13 is shown as $$\gamma_n = \frac{H}{V}\int \left(\Phi^{\langle n\rangle}\right)^2 dR = \frac{\pi H a^2}{V}\sum_{k=1}^{10}\sum_{j=1}^{10} C_k^{\langle n\rangle}C_k^{\langle n\rangle}B_{kj}$$ It should read as $$\gamma_n = \frac{\pi H}{V}\int r\left(\Phi^{\langle n\rangle}\right)^2 dr = \frac{\pi H a^2}{V}\sum_{k=1}^{10}\sum_{j=1}^{10} C_k^{\langle n\rangle}C_j^{\langle n\rangle}B_{kj}$$
17	Equation A.14 is shown as $$H_n = -H + \frac{a}{\lambda_n}\left(1 + 2\frac{b_n}{h_n}\right)$$ The second term is incorrect. The equation should read as follows. $$H_n = -H + \left(H - H_{cg}\right)\frac{h_n}{b_n} + \frac{a}{\lambda_n}$$ where H_{cg} is the height of the liquid CG relative to the origin (tank bottom in this case).
18	Equation A.15 is shown as $$h_n = \frac{2a\pi}{HV\lambda_n}\int ZR\Phi^{\langle n\rangle}dZ = \frac{2\pi a^4}{V\gamma_n\lambda_n H}\sum_{k=1}^{10} C_k^{\langle n\rangle}\bar{h}_k$$ It should read $$h_n = \frac{2a\pi}{\left(H - H_{cg}\right)V\gamma_n\lambda_n}\int zr\Phi^{\langle n\rangle}dz = \frac{2\pi a^4}{\left(H - H_{cg}\right)V\gamma_n\lambda_n}\sum_{k=1}^{10} C_k^{\langle n\rangle}\bar{h}_k$$ Where H_{cg} is the height of the liquid CG relative to the origin (tank bottom in this case).
19	The expression for Ω in Equation 4.38 is shown as $$\Omega = \frac{\omega}{\sqrt{(1 + Bo)(\sigma R_0^3/\rho)}}$$ This expression should read as $$\Omega = \frac{\omega}{\sqrt{(1 + Bo)(\sigma/R_0^3\rho)}}$$

20	Equation 4.46a is shown as $$\omega_1 = 1.62\left[\frac{\sigma}{\rho R_0^2}(1+0.798Bo)\right]^{1/2}$$ Equation 4.46a should read as $$\omega_1 = 1.62\left[\frac{\sigma}{\rho R_0^3}(1+0.708Bo)\right]^{1/2}$$
21	In the bullet list at the bottom of page 103, the value for the hydrazine surface tension is listed as σ=60 dyne/in This value should be shown as σ=60 dyne/cm
22	In Figure 3.2, p. 44, the tank width is defined as "2a". This should be changed to "a" to be consistent with derivations in Chapter 1 and the slosh model parameters in Table 3.1.
23	On p. 44, in the paragraph next to Figure 3.2, the sentence "The width of the tank is 2*a*" should read "The width of the tank is *a*".
24	The arguments of the hyperbolic functions in Table 3.1. p. 47, and in various equations in Chapter 3 are not clear. Change → To: $\tanh(2n-1)\,\pi h/a$ → $\tanh\left[\pi(2n-1)\frac{h}{a}\right]$ $\sinh(2n-1)\,\pi h/a$ → $\sinh\left[\pi(2n-1)\frac{h}{a}\right]$ $\tanh(2n-1)\,\pi h/2a$ → $\tanh\left[\pi(2n-1)\frac{h}{2a}\right]$
25	On p. 46, there is a missing "i" in the denominator of the left sides of Equations 1.28 and 1.34 (these should be identical to the equations of the same number in Chapter 1 on p. 10 and p. 12, respectively. The left side of Eq. 1.28 reads $$\frac{F_{xo}}{-\Omega^2 X_o m_{liq}}$$ To be consistent with the same equation ion Ch. 1, it should read $$\frac{F_{xo}}{-i\Omega^2 X_o m_{liq}}$$ The left side of Eq. 1.34 reads $$\frac{-F_{xo}}{-\Omega^2 X_o m_{liq}}$$ To be consistent with the same equation ion Ch. 1, it should read $$\frac{-F_{xo}}{i\Omega^2 X_o m_{liq}}$$

26	On p. 47, Equation 3.12 is incorrect. It reads as $$\frac{H_n}{h} = \frac{1}{2} - \frac{a}{2h}\frac{\tanh[(2n-1)\pi h/2a]}{(2n-1)\pi}$$ The correct equation is $$\frac{H_n}{h} = \frac{1}{2} - \frac{2a}{h}\frac{\tanh\left[(2n-1)\pi\frac{h}{2a}\right]}{(2n-1)\pi}$$
27	In Equation 3.5, p. 45, the lower case h_n should be replaced with upper case H_n.
28	In Table 3.1, the “Pendulum hinge position, H_n” unfortunately uses the same symbol as the mass position H_n, of the spring-mass model that is used in Eq. 3.5, 3.6, 3.7, 3.8, and 3.9. DO NOT CONFUSE THE H_n OF TABLE 3.1 WITH THE MASS POSITION OF THE SPRING MASS MODEL. The variable “$H_{hinge,n}$” should be used in this row of the tables. Add a row to the table to explicitly show the mass position, H_n. Mass Position, H_n : $\frac{H_n}{h} = \frac{1}{2} - \frac{2a}{h}\frac{\tanh\left[(2n-1)\pi\frac{h}{2a}\right]}{(2n-1)\pi}$
29	On p. 47, in the “Summary” paragraph, the last line has the incorrect equation $L_n = (g/\omega_n)^{0.5}$. The correct equation is $L_n = g/(\omega_n^{0.5})$
30	The identities near the bottom of p. 46 are incorrect. The first of these reads $$\frac{1}{\omega_n^2 - \Omega^2} = 1 + \frac{\Omega^2}{\omega_n^2 - \Omega^2}$$ The correct equation is $$\frac{\omega_n^2}{\omega_n^2 - \Omega^2} = 1 + \frac{\Omega^2}{\omega_n^2 - \Omega^2}$$ The second of these identities is nonsense. The intended identity is probably that of Eq. 3.10, $$\frac{m_n}{m_{liq}} = 8\left(\frac{a}{h}\right)\frac{\tanh\left[\pi(2n-1)\frac{h}{a}\right]}{[\pi(2n-1)]^3}$$ When this is substituted into Eq. 1.34, the analogy between the terms inside the summation and the corresponding terms of Eq. 3.9 can be made to define the mass position, H_n, as in Eq. 3.12.
31	OBSERVATION FOR LATER CHECKING In Figure 5.2, p. 119, the tank width is defined as “2a”. Ch 5 equations look like they are based on width = 2a. This is different than in Chapters 1 and 2.